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Fast jitter traveling wave solutions in a soliton communication line system with dispersion management

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Abstract

We derive traveling wave solutions for a communication system line with dispersion management using a perturbative approach. Use of conservation laws then show that a fast jitter happens when an initial frequency correction is included. © 1997 Published by Elsevier Science B.V.

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In recent times, research on the behavior of long communication lines has taken center stage due to the demand to improve the existing long communication systems. Of particular interest for both soliton or return to zero (RZ) and non-return to zero (NRZ) systems are lines with dispersion management, where the dispersion coefficient changes between amplifiers in a prescribed fashion [1,2]. The modeling of this system gives to first approximation a non-linear Schrödinger equation (NLSE), with the dispersion (due to management) and the non-linear coefficient (due to amplification) functions of the propagation variable z. Furthermore, due to the scalings in the equation, there are dispersion schemes in long communication lines for which the amplifier separation distance Z_a is much smaller that the dispersion distance Z_d. Here then, the coefficients in the governing equation are rapidly varying functions of Z. To generate useful information from this equation, more analysis is required. In recent works [3–7], averaged equations of the NLSE type have been derived, by several non-trivial methods, for different dispersion management schemes. The analysis also provides the first correction on the envelope of the electromagnetic field, which is of small amplitude but rapidly varying, as well as the first correction on the averaged NLSE, which appears as a perturbation in a similar fashion as in many other optical systems (see e.g. Ref. [8]).

As stated before, the analysis leading to an averaged equation to what would then be the averaged envelope (average here means after the propagation through a large number of amplifiers) is not a regular perturbation expansion and is very powerful in that it does not restrict to a particular solution form. Here, by setting a slightly more modest goal, that of finding traveling wave solutions, we are able to work on the original

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equation. We find these solutions by using conservation laws and by expending the solution we obtain the leading order and first correction using standard perturbation methods. Let us emphasize from the start, that by working on the original equation, these traveling wave solutions show to first order a rapid variation in z, which cannot be captured if we average the equations at the start. We then show that the average on the solutions we obtain, correspond to those of the averaged equations, thus validating the procedure we used. Although this approach works only for the traveling wave solutions, these are the most relevant solutions for applications (the more predominant being the soliton if the average of the dispersion coefficient is positive or the continuous wave if this average is zero).

The model considered here is governed by the dimensionless equation [3]

$$\mathrm{i}\frac{\partial q}{\partial Z} + \tfrac{1}{2}d(Z/\epsilon)\frac{\partial^2 q}{\partial T^2} + |q|^2 q = -\mathrm{i}\Gamma q + \mathrm{i}G(Z/\epsilon)q, \tag{1}$$

where Z_a is the amplifier spacing which is much smaller than the dispersion length Z_d, (i.e. $Z_a/Z_d = \epsilon \ll 1$). Here we will assume the dispersion management scheme to be periodic [1],

$$\begin{aligned} d(Z/\epsilon) &= d_1, \quad (n-1) < Z/\epsilon < n, \\ &= d_2, \quad n < Z/\epsilon < (n+1), \end{aligned} \tag{2}$$

thus we define the average over the period $Z_p/\epsilon = 2$ as

$$\langle f\rangle = \tfrac{1}{2}\int_s^{s+2} f(.,z)\,\mathrm{d}z, \quad z = Z/\epsilon. \tag{3}$$

Following the standard procedure, we make the transformation

$$q(T,Z) = a(Z)u(Z,T), \tag{4}$$

where

$$\mathrm{d}a/\mathrm{d}Z = -[\Gamma - G(Z/\epsilon)]a. \tag{5}$$

The scaled envelope u then satisfies

$$\mathrm{i}\frac{\partial u}{\partial Z} + \tfrac{1}{2}d(Z/\epsilon)\frac{\partial^2 u}{\partial T^2} + a^2(Z/\epsilon)|u|^2 u = 0. \tag{6}$$

It is at this stage that either a Lie transform method [3], or a stationary phase approximation [7] (which applies if $\langle d\rangle = 0$ or small) has been used to derive averaged equations that would eliminate the fast dependence of the dispersion and non-linear coefficient in (6). Breathing solitons have been obtained in the regime where the characteristic dispersion distance is of the same order as the separation between amplifiers [5], where the usual guiding center soliton does not apply. Here we point out a general feature of traveling waves independent of the particular regime. We do so, by recognizing some conservation laws of (6). In fact,

$$P = \int_{-\infty}^{\infty} |u|^2\,\mathrm{d}T, \tag{7}$$

$$M = \frac{\mathrm{i}}{2}\int_{-\infty}^{\infty}\left(u\frac{\partial u^*}{\partial T} - \mathrm{cc}\right)\mathrm{d}T, \tag{8}$$

are exact conserved quantities of Eq. (6). Two other quantities of interest are the center of mass

$$\bar{T} = \int_{-\infty}^{\infty} T|u|^2\,\mathrm{d}T \tag{9}$$

and the Hamiltonian

$$H(u,u^*,Z/\epsilon) = \int_{-\infty}^{\infty}\left(\frac{a^2(Z/\epsilon)}{2}|u|^4 - d(Z/\epsilon)\left|\frac{\partial u}{\partial T}\right|^2\right)\mathrm{d}T. \tag{10}$$

The center of mass satisfies the equations

$$\mathrm{d}\bar{T}/\mathrm{d}Z = Md(Z/\epsilon), \tag{11}$$

whose solution is

$$\bar{T}(Z/\epsilon) = \epsilon M\int^{Z/\epsilon} d(s)\,\mathrm{d}s + T_0.$$

The Hamiltonian on the other hand satisfies

$$\frac{\mathrm{d}H}{\mathrm{d}Z} = \frac{1}{\epsilon}\frac{\partial H}{\partial z}, \tag{12}$$

where $z = Z/\epsilon$. A consequence of the conservation of momentum and the equation for the center of mass $\bar{T}$, is that Eq. (6) also possesses a Galilean "boost". That is, if $v(T,Z)$ is any solution of (6), so is

$$u = v(Z, T - \bar{T}) \exp[i\phi(Z/\epsilon) + i\kappa T], \tag{13}$$

where

$$d\phi/dz = -\tfrac{1}{2}\kappa^2 d(z), \quad d\bar{T}/dz = \kappa d(z), \quad z = Z/\epsilon, \tag{14}$$

This result is valid for any functions $d(z)$ and results from the invariants of the original (not the averaged) equation.

We can now apply (13), (14) to any solution derived in Refs. [3–7]; instead we restrict our discussion for traveling wave solutions, with particularly relevant dispersion schemes. In particular, the breathing solitons found in Ref. [5] can be modified according to Eqs. (13), (14).

We proceed now to construct traveling wave solutions by first finding stationary solutions and then applying the Galilean boost, as an alternative of the use of the Lie transform method. To do so, we assume a solution of Eq. (6) of the form

$$u(T,Z,z) = [f(T) + \epsilon u_1(T,Z,z) + \epsilon^2 u_2(T,Z,z) + O(\epsilon^3)] \exp(i\lambda^2 Z), \tag{15}$$

where $u_1 = u_{1R} + i u_{1I}$. Substituting (15) in Eq. (6) gives to first order, for the real part

$$\partial u_{1R}/\partial z = 0, \tag{16}$$

and for the imaginary part

$$-\lambda^2 f + \frac{d(z)}{2}\frac{d^2 f}{dt^2} + a^2(z) f^3 = \frac{\partial u_{1I}}{\partial z}. \tag{17}$$

The solution of Eq. (16) is simply $u_{1R} = 0$, whereas for (17), we first impose the solvability condition that the right-hand side is orthogonal to the solutions of $\partial u_{1I}/\partial z = 0$, which simply translate into demanding that the average of the r.h.s. of Eq. (17) be zero. This gives the equation for f,

$$-\lambda^2 f + \tfrac{1}{2}\langle d\rangle \frac{d^2 f}{dT^2} + \langle a^2\rangle f^3 = 0, \tag{18}$$

and now after integrating (17),

$$u_{1I}(T,z) = \left(\int^{z} \tfrac{1}{2}\tilde{d}(s)\,ds\right)\frac{d^2 f}{dT^2} + \left(\int^{z} \tilde{a}(s)\,ds\right) f^3 + g_1(T), \tag{19}$$

where $\tilde{d} = d - \langle d\rangle$ and $\tilde{a} = a^2 - \langle a^2\rangle$. Following this procedure to $O(\epsilon)$ we find that

$$\partial u_{2R}/\partial z = -\tfrac{1}{2} d(z)\,\partial^2 u_{1I}/\partial T^2 - a^2(z) f^2 u_{1I}, \tag{20}$$

$$\partial u_{2I}/\partial z = 0. \tag{21}$$

In this case, $u_{2I} = 0$ and the solvability condition that the average of the right-hand side of (20) is zero, gives an equation for $g_1(T)$ and

$$u_{2R} = \int^{z} h_1(T,s)\,ds, \tag{22}$$

where $h_1(T,z)$ is the right-hand side of Eq. (20).

In the remainder of this Letter, we discuss the significance of these results and in particular, we make a comparison with what results using the guiding center soliton equations. Beginning with Eq. (17), this second-order equation is the same one obtains from the traveling wave ansatz for the integrable NLSE. The solutions are well known; if $\langle d\rangle > 0$, the solutions are T periodic waves and the homoclinic orbit corresponding to the soliton,

$$f(T) = \sqrt{2/\langle a^2\rangle}\,\lambda\,\mathrm{sech}(\sqrt{2/\langle d\rangle}\,\lambda T). \tag{23}$$

Given any solution of Eq. (18) and applying the Galilean boost (13), (14), we obtain traveling wave solutions, in particular, the soliton-like pulse

$$u_0(T,Z,Z/\epsilon) = \sqrt{2/\langle a^2\rangle}\,\lambda \times \mathrm{sech}\left(\sqrt{\frac{2}{\langle d\rangle}}\lambda[T - \bar{T}(Z/\epsilon)]\right) \times \exp[i\lambda^2 Z + i\phi(Z/\epsilon) + i\kappa T]. \tag{24}$$

If $\langle d\rangle < 0$, then the traveling wave solutions are either periodic or the dark soliton-like

$$u_0(T,Z,Z/\epsilon) = (\lambda/\sqrt{\langle a^2\rangle}) \times \tanh\left(\frac{\lambda}{\sqrt{-\langle d\rangle}}[T - \bar{T}(Z/\epsilon)]\right) \times \exp[i\lambda^2 Z + i\phi(Z/\epsilon) + i\kappa T]. \tag{25}$$

In the last case $\langle d\rangle = 0$, Eq. (18) becomes algebraic thus producing traveling cw solutions of the form

$$u_0(T,Z,Z/\epsilon) = (\lambda/\sqrt{\langle a^2\rangle}) \times \exp[\mathrm{i}\lambda^2 Z + \mathrm{i}\phi(Z/\epsilon) + \mathrm{i}\kappa T]. \quad (26)$$

In all cases $\phi, \bar{T}$ satisfy Eq. (14).

Notice that these solutions are different from the leading-order solutions of the equations that result after applying the Lie transform method, which we now define as $\bar{u}_0$. This is true because (24)-(26) have an explicit Z/ϵ dependence, which is not the case in $\bar{u}_0$. On the other hand, it turns out that

$$f(T-\langle\bar{T}\rangle)\exp(\mathrm{i}\lambda^2 Z + \mathrm{i}\langle\phi\rangle + \mathrm{i}\kappa T) = \bar{u}_0, \quad (27)$$

which effectively states that the "average" of the rapidly varying solution of (6) is equal to the solution of the "averaged" (i.e. Lie transformed) guiding center equation. This correspondence between the two approaches was also pointed out in Ref. [9], in their study of phase sensitive amplifiers in a communication system.

If we follow up the correspondence between the two methods to higher orders, we find that $u_1 = \mathrm{i}u_{11}$ given by Eq. (19), is exactly the same as the first-order correction found in Ref. [3] (where it is defined as ϕ_1). Finally we go back to Eq. (6) and write it as

$$-\lambda^2 f + \tfrac{1}{2}d(z)\frac{\mathrm{d}^2 f}{\mathrm{d}T^2} + a^2(z)f^3 = -\mathrm{i}\frac{\partial u_1}{\partial z} + \epsilon\lambda^2 u_1 - \mathrm{i}\epsilon\frac{\partial u_2}{\partial z} + \mathrm{O}(\epsilon^2), \quad (28)$$

where right-hand side can be evaluated in terms of f, from Eqs. (17), (19), (20). Notice that the right-hand side is now a function of $f(T)$ and its derivatives and of $z = Z/\epsilon$ as well. Then, by taking the average of Eq. (28) (notice in particular that $\langle\partial u_1/\partial z\rangle = 0$, $\langle u_1\rangle = \mathrm{i}g$, and $\langle\partial u_2/\partial z\rangle = \langle\partial u_{2\mathrm{R}}/\partial z\rangle$), we obtain a perturbed ode for $f(T)$, which is the traveling wave equation corresponding to the extended NLSE derived by the Lie transform method (see Eq. (30) in Ref. [3] and Eqs. (33a), (33b) in Ref. [6]).

In conclusion, we have derived traveling wave solutions of optical pulses in a dispersion managed communication line using multiple scale perturbation methods. By exploiting conservation laws and the Galilean boost we find that these solutions capture fast variations in the pulse centroid and phase. We then observe that by averaging these two quantities, one recovers the results obtained after applying the Lie transform methods. The method used here is simpler, at the minor cost of reducing the problem to one of seeking traveling wave solutions. More general solutions can only be found using Ref. [3]. But the results include most of the relevant solutions and gives further evidence of the validity of the Lie transform method, applied to infinite dimensional partial differential equations [10]. This is only a formal approach in that there is no proof that the method would converge to solutions of the original equation. This work, although also not rigorous, indicates that effectively, for traveling wave solutions, the average of the solutions of (6) is equal to the solution of the averaged equation.

We do not know at this time of any possible consequences of this fast jitter; for example it would be of interest to see the effect on pulse collisions given the recent work by Wabnitz [11] where he shows that periodic dispersion compensation reduces the effects of resonant collisions in soliton wave division multiplexing transmissions. It is interesting to point out that for the dispersion management system governed by Eq. (6), we find that the mean square of the timing jitter for long propagation distances satisfies to leading order,

$$\langle\bar{T}^2(Z)\rangle = \tfrac{2}{3}\mu\langle d\rangle^2 Z^3, \quad (29)$$

where μ is proportional to the mean square frequency shift due to the effects of the amplifiers. Eq. (29) is the same as in the case of uniform dispersion $d = \langle d\rangle$ [12]. In both cases it is assumed that the initial frequency $\kappa = 0$.

An important issue is to see if this equivalence follows when additional perturbations arise; this we intend to study in the near future. Understanding the behavior of solutions in resonance to the period of the coefficients in (6) also remains an open problem. Finally, it is interesting to observe the dramatic difference in the form of the corrections when compared to the other extreme in which the coefficients in (6) are slowly varying (i.e. functions of ϵz), where soliton perturbation theory applies [13].

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References

[1] A.H. Gnauck, A.R. Chaplyvy, R.W. Tkach and R.M. Berosier, IEEE Elec. Lett. 31 (1995) 277.
[2] N.S. Bergano and C.R. Davidson, OFC'96 PD23, San Diego, CA (1996).
[3] A. Hasegawa and Y. Kodama, Opt. Lett. 16 (1991) 1385.
[4] Y. Kodama and S. Wabnitz, Opt. Lett. 20 (1995) 2291.
[5] I.R. Gabitov and S. Turitsyn, Opt. Lett. 21 (1996) 327; Pis'ma Zh ETF 63 (1996) 814.
[6] J.C. Bronski and J.N. Kutz, Guiding-center pulse dynamics in non-return to zero (return-to-zero) communications systems with mean-zero dispersion, J. Opt. Soc. Am. B (1996), to appear.
[7] J.C. Bronski and J.N. Kutz, Asymptotic behavior of the nonlinear Schrödinger equation with rapidly varying, mean zero dispersion, to appear in Physica D (1997).
[8] S. Wabnitz, Y. Kodama and A.B. Aceves, Opt. Fiber Tech. 1 (1995) 187.
[9] J.N. Kutz, C.V. Hille, W.L. Kath, R.D. Li and P. Kumar, J. Opt. Soc. Am. B 11 (1994) 2212.
[10] Y. Kodama, Phys. Lett. A 112 (1985) 193.
[11] S. Wabnitz, Opt. Lett. 21 (1996) 638.
[12] J.P. Gordon and H.A. Haus, Opt. Lett. 11 (1986) 665.
[13] S. Kumar and A. Hasegawa, Opt. Lett. 21 (1996) 1903.